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# A Study on Red Shift and Newtonian Analogue of Force for a Cosmological Model with Electromagnetic Field.

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**Abstract:**

The paper presents an investigation on relativistic red shift and Newtonian analogue of force for a cylindrically symmetric cosmological model which is derived for a perfect fluid with an electromagnetic field. Here we have shown that the force terms Ri and Si both are null force vectors and $R_4$ and $S_4$ have no Newtonian analogues. We have also found these terms in absence of electromagnetic field.

**Keywords**: Red shift, Newtonian analogue of force, perfect fluid, model, light pulse.

## 1. INTRODUCTION :

Roy and Prakash [5] have discussed the red shift (Doppler effect) and behavior of a test particle by taking an anisotropic magneto hydrodynamic cosmological model which they derived taking cylindrically symmetric metric of Marder [2] in presence electromagnetic field. The work has been further extended by Singh and Yadav [6] and then by Yadav and Purushottam [8] for non static cylindrically symmetric cosmological model which is spatially homogeneous non degenerate Petrov type I. Further by taking a suitable metric Yadav et al. [9] have studied red shift and behavior of a test particle in general relativity.

Here this paper provides an investigation on Dopper effect (red shoft) and Newtonian analogue of force for a cylindrically symmetric non-static cosmological model with electromagnetic field. This model has been obtained by me in an earlier paper [1] and is given by

$$(1.1)\quad ds^2 = dT^2 - dx^2 - (T^2-\phi^2)^{1/2}[T+(T^2-\phi^2)^{1/2}]^{\alpha} dY^2 - (T^2-\phi^2)^{1/2}[T+(T^2-\phi^2)^{1/2}]^{-\alpha} dZ^2$$

where for real metric $T^2 > \phi^2$ and symbols have same meanings as given in paper [1]. Here we have also made study for above investigations when there is no electromagnetic field.

## 2. The Doppler effect (Red Shift) in the Model

The tract of light pulse in the model (1.1) is given by setting $ds^2 = 0$. i.e.

$$(2.1)\quad \left(\frac{dX}{dT}\right)^2 + (T^2-\phi^2)^{1/2}\left[T+(T^2-\phi^2)^{1/2}\right]^{\alpha}\left(\frac{dY}{dT}\right)^2 + (T^2-\phi^2)^{1/2}\left[T+(T^2-\phi^2)^{1/2}\right]^{-\alpha}\left(\frac{dZ}{dT}\right)^2 = 1$$

For the case when velocity is along z-axis, equation (2.1) gives

(2.2) $$\frac{dZ}{dT} = \pm(T^2-\phi^2)^{-1/4}\left[T+(T^2-\phi^2)^{1/2}\right]^{\alpha/2} = \pm\psi(T)$$

Hence the light pulse leaving a particle at (0, 0, z) at time $T_1$ would arrive at a later time $T_2$ given by

(2.3) $$\int_{T_1}^{T_2}\psi(T)dT = \int_0^Z dZ$$

Hence,

$$\psi_2(T)\delta T_2 = \psi_1(T)\delta T_1 + \frac{dZ}{dT}\delta T_1$$
$$= \psi_1(T)\delta T_1 + U_z\delta T_1$$

where $\left(\frac{dz}{dT}\right) = U_z$ is the z-component of the velocity of he particle at the time of emission and $\psi_1(T)$ and $\psi_2(T)$ are the values of $\psi(T)$ for T = $T_1$ and T = $T_2$ respectively. From the above equation we get

(2.4) $$\delta T_2 = \left\{\frac{\psi_1(T)+U_z}{\psi_2(T)}\right\}\delta T_1$$

The proper time interval $\delta T_1^0$ between successive wave crests as measured by the local observer moving with the source is given by

(2.5) $$\delta T_1^0 = \left\{1-\left(\frac{dX}{dT}\right)^2 - (T^2-\phi^2)^{1/2}\left[T+(T^2-\phi^2)^{1/2}\right]^{\alpha}\right.$$
$$\left.\left(\frac{dY}{dT}\right)^2 - (T^2-\phi^2)^{1/2}\left[T+(T^2-\phi^2)^{1/2}\right]^{-\alpha}\left(\frac{dZ}{dT}\right)^2\right\}^{1/2}\delta T_1$$

This can be written as

(2.6) $$\delta T_1^0 = \left\{1-U^2\right\}^{1/2}\delta T_1$$

where U is the velocity of source at the time of emission. Simlarly we may write

(2.7) $$\delta T_0^2 = \delta T_2$$

as the proper time interval between the reception of two successive wave crests by an observer at rest at the origin. Hence following Tolman [7] the red shift in this case is given by

(2.8) $$\frac{\lambda+\delta\lambda}{\lambda} = \frac{\delta T_2^0}{\delta T_1^0} = \frac{\left\{(T_1^2-\phi^2)^{1/4}\left[T_1+(T_1^2-\phi^2)^{1/2}\right]^{\alpha/2}+U_z\right\}}{\left\{(T_2^2-\phi^2)^{-1/4}\left[T_2+(T_2^2-\phi^2)^{1/2}\right]^{\alpha/2}\right\}\left\{1-U^2\right\}^{1/2}}$$

## 3. Newtonian Analogue of Force in the Model

Here we study the effect of electromagnetic field in force terms $R_i$ and $S_i$ (Narlikar and Singh [3]). It is well known that Christoffel three index symbols for the Riemannian metric of general relativity.

(3.1) $ds^2 = g_{ij}\, dx^i\, dx^j$

defined by

(3.2) $$[i,j,k] = \frac{1}{2}\left[\frac{\partial g_{ik}}{\partial x^j} + \frac{\partial g_{jk}}{\partial x^i} - \frac{\partial g_{ij}}{\partial x^k}\right]$$

and

(3.3) $$\Gamma^k_{ij} = g^{kl}[ij,l]$$

are not tensors, but the difference of two such symbols of the same kind is a tensor. Thus if $\left\{ {i \atop j\,k} \right\}$ be Christoffel three-index symbol for another metric

(3.4) $$ds^2 = Y_{ij} dx^i dx^j$$

the difference

(3.5) $$\Gamma^i_{jk} - \left\{ {i \atop j\,k} \right\} = \Delta^i_{jk}$$

is a tensor

The equation of the geodesix may now be cast in the form

(3.6) $$\frac{d^2x^i}{ds^2} + \Gamma^i_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds} = -\Delta^i_{jk}\frac{dx^i}{ds}\frac{dx^k}{ds}$$

In a region free from gravitation the geodesics are given by

(3.7) $$\frac{d^2x^i}{ds^2} + \Gamma^i_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds} = 0$$

Hence the gravitational effects depend only on the residual term $-\Delta^i_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds}$. Since $\left(\frac{dx^i}{ds}\right)$ is arbitrary, the gravitational effects have to be associated with the tensor $\Delta^i_{jk}$ only. It can be seen that Rosen [4]

(3.8) $$\Delta^i_{jk} = \frac{1}{2}g^{il}\left(g_{jl,k} + g_{kl,j} - g_{jk,l}\right)$$

Where Comma (,) denotes co-variant differentiation with respect to Y-metric. A co-variant differentiation with respect to g-metric will be denoted by a semi colon (;)

From (3.8) we have

(3.9) $$g_{im}\Delta^i_{jk} = \frac{1}{2}\left[g_{jm,k} + g_{km,j} - g_{jk,m}\right]$$

If we write

(3.10) $g_{im}\Delta^{i}_{jk} = \Delta_{jkm}$

Then $\Delta^{i}_{jk}$ and $\Delta_{ijk}$ as given by (3.8) and (3.9) are associated tensors.

The $\Delta$ - tensor thus defined contain only the first order partial derivatives of the metric potential $g_{ij}$. Since $\Delta$-tensors are defined against the background of an arbitrary flat substratum, they would vary with the choice of the later for the same gravitational field. The importance of the $\Delta^{s}$ consists in the fact that the gravitational force of the Newtonian theory appears through them.

The vectors $R_i$ and $S_i$ are defined as follow (Narlikar and Singh [13]

(3.11) $R_i = \Delta^{j}_{ji} = \dfrac{\tau_{,i}}{\tau}$

(3.12) $S_i = \Delta^{\ell}_{jk} g^{jk} g_{\ell i}$

$$= g^{jk} g_{ji,k} - \frac{\tau_{,i}}{\tau}$$

where $\tau = \sqrt{\dfrac{g}{Y}}$

For the line element (3.8), we have

$$(3.13)\quad \begin{cases} g_{11} = -1 \\ g_{22} = -(T^2-\phi^2)^{1/2}\left[T+(T^2-\phi^2)^{1/2}\right]^{\alpha} \\ g_{33} = -(T^2-\phi^2)^{1/2}\left[T+(T^2-\phi^2)^{1/2}\right]^{-\alpha} \\ g_{44} = 1 \end{cases}$$

and

$$(3.14)\quad \begin{cases} g^{11} = -1 \\ g_{22} = -(T^2-\phi^2)^{-1/2}\left[T+(T^2-\phi^2)^{1/2}\right]^{-\alpha} \\ g_{33} = -(T^2-\phi^2)^{-1/2}\left[T+(T^2-\phi^2)^{1/2}\right]^{\alpha} \\ g_{44} = 1 \end{cases}$$

(3.15) $g = -\left(T^2 - \phi^2\right)$

The corresponding flat metric $Y_{ij}$ is taken to be that of special relativity

(3.16) $ds^2 = dT^2 - dX^2 - dY^2 - dZ^2$

Thus

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(3.17) $Y_{ij} = [-1,-1,-1,1]$

(3.18) $Y = -1$

From equations (3.15) and (3.18)

(3.19) $\tau = \sqrt{\frac{g}{Y}} = \left(T^2 - \phi^2\right)^{1/2}$

From (3.11) and (3.12) we get

(3.20) $R_i = [0,0,0,T(T^2 - \phi^2)^{-1}]$

and

(3.21) $S_i = [0,0,0,-T(T^2 - \phi^2)^{-1}]$

Thus we find that $R_i$ and $S_i$ both are null force vectors. $R_4$ and $S_4$ have no Newtonian analogues.

In the absence of electromagnetic field the model is given by the metric

(3.22) $ds^2 = dT^2 - dX^2 - \frac{1}{2}(2T)^{\alpha+1}dY^2 - \frac{1}{2}(2T)^{-\alpha+1}dZ^2$

Therefore the vectors $R_i$ = and $S_i$ reduce to

(3.22) $R_i = \left[0,0,0,\frac{1}{T}\right]$

and

(3.23) $s_i = \left[0,0,0,-\frac{1}{T}\right]$

The red shift in the model is given by

(3.25) $\frac{\lambda + \delta\lambda}{\lambda} \frac{\left[(2)^{\alpha/2}T_1^{(\alpha-1)/2} + U_Z\right]}{\left[(2)^{\alpha/2}T_2^{(\alpha-1)/2} + (1-U^2)^{1/2}\right]}$

where

$$U_Z = \frac{dZ}{dT} = (2)^{\alpha/2}(T)^{(\alpha-1)/2}$$

is the velocity at the time of emission.

## 4. Conclusion

In this paper we found and discussed Dopper effect and Newtonian analogue of for the considered model with electromagnetic field given by (1.1). We have found that $R_i$ and $S_i$ both are null force vectors. Also $R_4$ and $S_4$ have no Newtonian analogues. Further in absence of electromagnetic field the model is given by (3.22). In this case we have also obtained Newtonian analogue of force and red shift given by (3.25).

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